Probabilistic Gathering of Agents with Simple Sensors

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1. Abstract

Novel probabilistic ***gathering*** algorithms for agents that can only detect ***the presence*** of other agents behind them.

Agents Properties:
- Identical and Indistinguishable
- Oblivious (have no memory)
- Have limited visibility
- No explicit communication
- No common frame of reference (GPS, compass)

The analysis of the gathering process assumes that the agents act synchronously in selecting random orientations that remain fixed during each unit time-interval.

2. Sensing and Dynamic-law

Sensing:
- On-Board Backward Looking Binary Sensor

$s_i(k)=\begin{cases}1, & \text{agent } i \text{ rear half plane does not contain agents}\\ 0, & \text{otherwise}\end{cases}$

Dynamics:
- All agents whose rear half plain does not contain other agents (e.g. $s_i(k)=1$) jump forward
- Then all the agents change their orientations by choosing a uniformly distributed random heading directions

3. Why “probabilistic”?

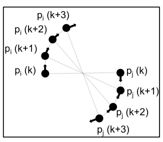


Figure 1. A rare case of consistent dispersion in two agents, due to "malicious" heading directions

4. Discrete time

Conditional forward jump – no agents behind

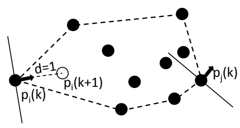
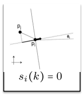


Figure 2. Agent *i* jumps a unit step since there are no agents behind it, while agent *j* stays put.

$$p_i(k+1) = p_i(k) + \begin{bmatrix}\cos(\theta_i(k))\\ \sin(\theta_i(k))\end{bmatrix} s_i(k)$$

$$\theta_i(k) = \sum_{k=1}^{\infty} \chi_k^{(i)} 1_{\Delta_k}(t)$$



where

$\chi_k^{(i)}$ are iid uniformly distributed over $[0, 2\pi]$

$$1_{\Delta_k}(t) = \begin{cases}1, & \text{for } t \in [k, k+1)\\ 0, & otherwise\end{cases}$$

$$s_i(k) = \begin{cases}0, & \exists\ j : \hat{\theta}_i^{\top}(k)[p_j(k) - p_i(k)] \le 0\\ 1, & otherwise\end{cases}$$

5. Simulations Results

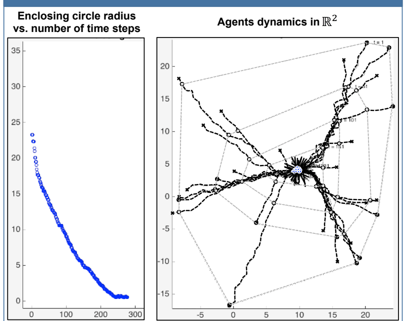


Figure 3. Simulation result on 30 agents, with initial random spread of 50 by 50 area and step-size 1.

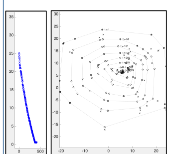


Figure 4. Simulation result on 40 agents

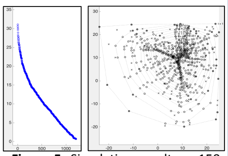


Figure 5. Simulation result on 150 agents

Convergence time vs. number of agents.
The effect of the number of agents on the convergence time is ***linear***.

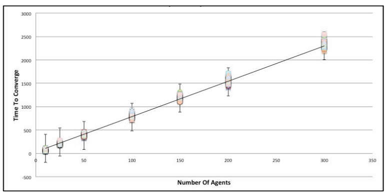


Chart 1. Convergence time vs. number of agents. Initial random spread of 50 by 50 area and step-size 1. This process ran for 75 repetitions with different random initial constellations.

6. Continuous time

Conditional move – no agents in sensing region

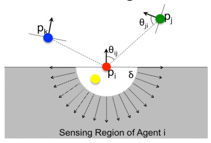
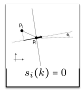


Figure 6. The dashed region of half-plane missing half-disc centered at p_i is the sensing coverage area of agent *i* with its dead-zone of radius δ. Agent *k* stays put while the others travel.

$$\dot{p}_i(t) = \begin{bmatrix}\cos(\theta_i(t))\\ \sin(\theta_i(t))\end{bmatrix} s_i(t)$$

$$\theta_i(t) = \sum_{k=1}^{\infty} \chi_k^{(i)} 1_{\Delta_k}(t)$$



where

$\chi_k^{(i)}$ are iid uniformly distributed over $[0, 2\pi]$

$$1_{\Delta_k}(t) = \begin{cases}1, & \text{for } t \in [k, k+1)\\ 0, & otherwise\end{cases}$$

$$s_i(t) = \begin{cases}0, & \exists\ j\ :\ d_{ij} > \delta \text{ and } \hat{\theta}_i^{\top}(t)[p_j(t) - p_i(t)] \le 0\\ 1, & otherwise\end{cases}$$

7. Formal proof

Theorem 1. Piece-wise continuous dynamics converges to a region of radius δ in finite expected time.
Principle of the proof:

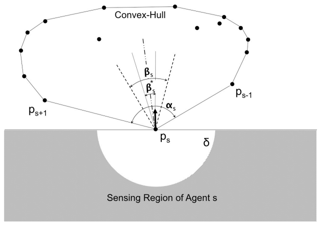


Figure 7. Agent s at the sharpest corner of the convex-hull is shown with its sensing area. Black arrow shows the selected heading direction.

$$\alpha_s \le \pi(1 - \frac{2}{n}) \longrightarrow Pr(\text{agent } s \text{ moves}) > \frac{1}{2n}$$

$$E(t)_{convergence} \le \frac{8n^3}{(1 - \sqrt{1 - \tan^2 \frac{\pi}{4n}})} \frac{d_{max}(0)}{\delta}$$

The “continuous version” ensures gathering to within a region of diameter 2δ.
Gathering happens in finite expected time, proportional to δ^{-1}, i.e. the blind spot of radius δ is absolutely necessary for finite expected time convergence. ■

8. Conclusions

Discrete time – found experimentally to gather the agents to a minimal enclosing circle of radius 1, in time proportional to the number of agents.
Continuous time – formal proof that the system converges to a region of radius δ in finite expected time.

Contact

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