1. Abstract

Novel probabilistic gathering algorithms for agents that can only detect the presence of other agents behind them.

Agents Properties:
- Identical and indistinguishable
- Oblivious (have no memory)
- Have limited visibility
- No explicit communication
- No common frame of reference (GPS, compass)

The analysis of the gathering process assumes that the agents act synchronously in selecting random orientations that remain fixed during each unit time-interval.

2. Sensing and Dynamic-law

Sensing:
- On-Board Backward Looking Binary Sensor
  \( s(k)^{(j)} \) \( \in \) \( \{0, 1\} \)
  - 1: agent i in half plane does not contain agents behind it
  - 0: otherwise

Dynamics:
- All agents whose rear half plane does not contain other agents (e.g. \( s(k)^{(i)} = 1 \)) jump forward
- Then all the agents change their orientations by choosing a uniformly distributed random heading directions

3. Why “probabilistic”?

A rare case of consistent dispersion in two agents, due to “nullious” heading directions

4. Discrete time

Conditional forward jump – no agents behind

\[ \begin{align*}
\theta_k &= \{0, 1\} \\
\mathbf{s}_k &= \{0, 1\} \\
\theta_{k+1} &= \begin{cases} 0 & \text{if } s_k = 0 \\
1 & \text{if } s_k = 1
\end{cases}
\end{align*} \]

Figure 2. Agent i jumps a unit step since there are no agents behind it, while agent j stays put.

5. Simulations Results

Convergence time vs. number of agents.

The effect of the number of agents on the convergence time is linear.

6. Continuous time

Conditional move – no agents in sensing region

\[ \begin{align*}
\dot{\theta}(t) &= \frac{1}{\delta^2} \mathbf{I} \mathbf{s}(t) \quad \text{if } \mathbf{s}(t) = 0 \\
\dot{\theta}(t) &= \frac{1}{\delta^2} \mathbf{I} \mathbf{-I} \quad \text{if } \mathbf{s}(t) \neq 0
\end{align*} \]

Figure 6. The dashed region of half plane ensuring half-disc centered at \( p \) is the sensing coverage area of agent \( i \) with its dead-zone of radius \( \delta \). Agent \( \delta \) stays put while the others travel.

7. Formal proof

Theorem 1. Piece-wise continuous dynamics converges to a region of radius \( \delta \) in finite expected time.

Principle of the proof:

Figure 7. Agent \( i \) at the sharpest corner of the convex-hull is chosen with its sensing area. Black arrow shows the selected heading direction.

\[ a_i \leq \mathbf{I} (1 - \frac{1}{2}) \rightarrow P(\text{agent } i \text{ moves}) \leq \frac{1}{2}\delta \]

\[ E(\text{convergence}) \leq \frac{8\delta^3}{(1 - \sqrt{1 - \tan^2(\frac{\delta}{2})})^2} \]

The “continuous version” ensures gathering to within a region of diameter 2\( \delta \).

8. Conclusions

Discrete time – found experimentally to gather the agents to a minimal enclosing circle of radius 1, in time proportional to the number of agents.

Continuous time - formal proof that the system converges to a region of radius \( \delta \) in finite expected time.

References